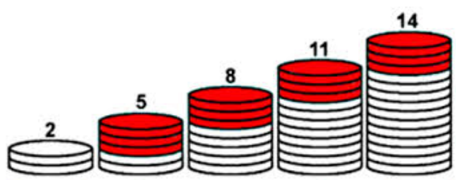
Warm Up

Find the next number in the sequence:

- $1, 2, 3, 4, \ldots$
- $5, 11, 17, \ldots$
- $1, 2, 4, 8 \ldots$
- $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

Arithmetic Sequences

An arithmetic sequence is a sequence such that every pair of adjacent terms differ by the same value. Some examples are the first and second bullet points in the Warm Up.



Arithmetic Sequence Notation

We can refer to a general arithmetic sequence using the following notation:

We let a be the first term of the sequence. Then, as each pair of adjacent terms differ by the same value, we can express that constant value as d. Hence, the first term is a, the second term is $a + d$, the third term is $a + 2d$, etc.

Arithmetic Sequence Exercises

1. Find the 10th term of the pattern $3, 5, 7, 9, \ldots$. What about the 100th? Or the 1000th? How would you find the nth term for any integer n?
2. Find the nth term of a general arithmetic sequence.
3. What's the sum of the first 4 terms of the pattern $1, 3, 5, 7, \ldots$? How about the first 7 terms? What about the first 11? Do you see a pattern?
4. Take an arithmetic sequence, say $5, 11, 17, 23, 29$. Reverse the terms, so now you have $29, 23, 17, 11, 5$. If you add up the corresponding terms (the two terms in the first position, two terms in the second position, and so on), what do you notice about the sums?
5. (**Challenge**) Find the sum of an arithmetic sequence with first term a, common difference d, and n terms in total.

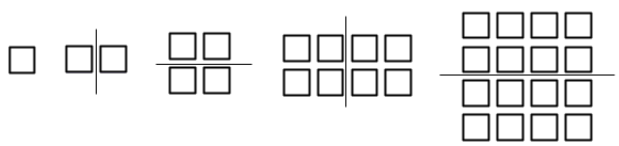
6. (2007 AMC 12A) Let a, b, c, d, and e be five consecutive terms in an arithmetic sequence, and suppose that $a + b + c + d + e = 30$. Which of a, b, c, d, or e can be found?

7. (**Challenge**) An arithmetic sequence is a sequence in which each term after the first is obtained by adding a constant to the previous term. Each row and each column in this 5×5 array is an arithmetic sequence with five terms. What is the value of X?

1				25
		X		
17				81

Geometric Sequences

An arithmetic sequence is a sequence such that when you divide a term by the previous term, you always get the same value. Some examples are the third and fourth bullet points in the Warm Up.



Geometric Sequence Notation

We can refer to a general geometric sequence using the following notation:

We let a be the first term of the sequence. Then, as each pair of adjacent terms have a ratio of the same value, we can express that constant value as r. Hence, the first term is a, the second term is ar, the third term is ar^2, etc.

Geometric Sequence Exercises

1. What is the 10th term of the geometric sequence $1, 2, 4, \ldots$? What about the 100th? 1000th? What about the nth term for any integer n?

2. Write out the first 6 terms of the geometric sequence $1, 3, 9, \ldots$. Multiply each of these 6 terms by 3, and the resulting 6 terms under the previous ones. What do you notice when you subtract these?

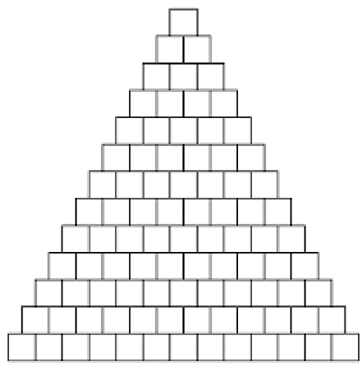
3. Write out the first 10 terms of the geometric sequence with starting term a and ratio r. Multiply each of these 10 terms by r, and write the resulting sequence underneath. What is remaining when you subtract these?

4. (**Challenge**) Let S be the sum of the first n elements of a general geometric series. Now take $r \times S$, and subtract S from it. Can you solve for S?
5. (**Challenge**) (2003 AMC 10B) The second and fourth terms of a geometric sequence are 2 and 6. Which of the following is a possible first term?

   (A) $-\sqrt{3}$ **(B)** $-\frac{2\sqrt{3}}{3}$ **(C)** $-\frac{\sqrt{3}}{3}$ **(D)** $\sqrt{3}$ **(E)** 3
6. (**Challenge**) Asher has a geometric sequence that starts with 3 and ends with 48. If each term is an integer, what are the different geometric sequences he could have come up with?

Pascal's Triangle

1. Fill in the top box with the number 1.
2. Fill the second line in with two 1's. For every line after, fill in the boxes on the sides with 1's.
3. Fill in each interior box with the sum of the values in the two boxes directly on top. For example, the third row will be {1, 2, 1} and the fourth row will be {1, 3, 3, 1}.



1. After filling out the triangle, what patterns in the numbers do you notice?
2. What if you were to take the sum of each row, what pattern do you see?
3. Now, color or shade in the boxes with odd values, do see anything familiar?
4. Challenge: Find a way to get the numbers of the Fibonacci sequence from Pascal's Triangle.

Pascal's Triangle can be used to find the number of combinations. Let's say that I have 5 marbles of different colors. On Pascal's triangle, the number of ways to choose a group of 2 of those marbles is 10, which is the 3rd entry of the 6th row.

1. Let's say you want to choose 3 students from a group of 4 students {Al, Bob, Carl, Dan}. What are the possible combinations of 3 students? Where do you find the number of combinations on Pascal's Triangle?
2. Now, say you want to choose 4 officers from a committee of 8 people? Without listing out the combinations, can you find the number of combinations from Pascal's Triangle?
3. **(Challenge)** Using the triangle, what is the probability of getting 6 heads from flipping a fair coin 11 times? What is the probability of getting **at least** 6 heads from flipping the coin 11 times?